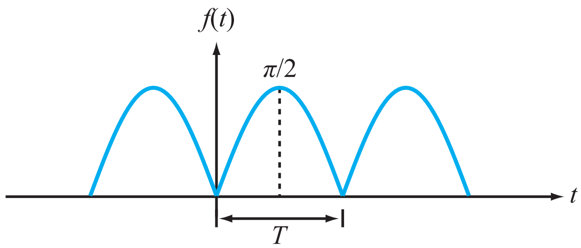
Problem 5.18 Repeat Problem 5.16 for $f(t) = |\sin(4\pi t)|$.

Solution:



Similar to Problem 5.16:

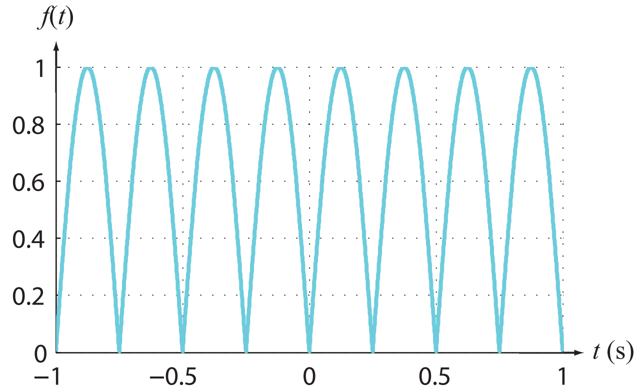
$$T = \frac{1}{4} \text{ s}$$

and $\omega_0 = (8\pi)$ rad/s.
$f(t)$ has even symmetry.

$$b_n = 0$$

$$\begin{aligned}
a_0 &= \frac{1}{T}\int_0^T |\sin(4\pi t)|\,dt \\
&= 4\int_0^{1/4} \sin(4\pi t)\,dt \\
&= -4 \times \left.\frac{\cos(4\pi t)}{4\pi}\right|_0^{1/4} \\
&= \frac{2}{\pi} \\
a_n &= \frac{2}{T}\int_0^T |\sin(4\pi t)|\cos(8\pi nt)\,dt \\
&= \frac{8}{2}\int_0^{1/4} \sin(4\pi t)\cos(8\pi nt)\,dt \\
&= 4\int_0^{1/4} [\sin(4\pi(1+2n)t) + \sin(4\pi(1-2n)t)]\,dt \\
&= -4\left.\left[\frac{\cos(4\pi(1+2n)t)}{4\pi(1+2n)} + \frac{\cos(4\pi(1-2n)t)}{4\pi(1-2n)}\right]\right|_0^{1/4} \\
a_n &= \frac{1-\cos(\pi(1+2n))}{\pi(1+2n)} + \frac{1-\cos(\pi(1-2n))}{\pi(1-2n)} = \frac{4\cos 8\pi nt}{\pi - 4n^2\pi}
\end{aligned}$$

$$\therefore \quad f(t) = \frac{2}{\pi} + \sum_{n=1}^{\infty}\left[\frac{1-\cos(\pi(1+2n))}{\pi(1+2n)} + \frac{1-\cos(\pi(1-2n))}{\pi(1-2n)}\right]\cos(8\pi nt)$$


f(t)
1
0.8
0.6
0.4
0.2
0
−1
−0.5
0
0.5
1
t (s)