Problem 5.25 The current source $i_s(t)$ in the circuit of Fig. P5.25 generates a train of pulses (waveform #3 in Table 5-4) with $A = 6$ mA, $\tau = 1$ μs, and $T = 10$ μs.

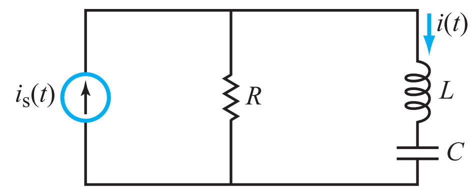
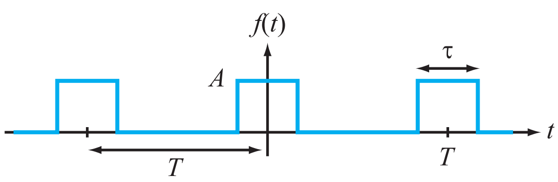


Figure P5.25: Circuit for Problem 5.25.

(a) Derive the Fourier series representation of $i(t)$.
(b) Calculate the first five terms of $i(t)$ using $R = 1$ kΩ, $L = 1$ mH, and $C = 1$ μF.
(c) Plot $i(t)$ and $i_s(t)$ using $n_{max} = 100$.

Solution:



(a) The train of pulses can be presented as

$$i_s(t) = \frac{A\tau}{T} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right) \cos\left(\frac{2n\pi t}{T}\right)$$

so that $\omega_0 = \frac{2\pi}{T} = 2\pi \times 10^5$ rad/s.

$$a_0 = \frac{A\tau}{T} = \frac{6 \times 10^{-3} \times 1 \times 10^{-6}}{10 \times 10^{-6}} = 6 \times 10^{-4}$$

$$a_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right)$$

$$b_n = 0$$

So in phasor domain

$$A_n \angle \phi_n = a_n - jb_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right).$$

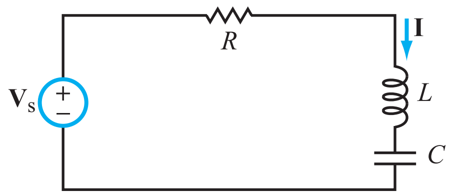
Hence

$$\mathbf{I}_s = \frac{A\tau}{T} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right).$$

Next we calculate the transfer function

$$\mathbf{H}(\omega) = \frac{\mathbf{I}(\omega)}{\mathbf{I}_s(\omega)}.$$

An equivalent circuit can be used to derive the expression for $H(\omega)$. Based on the Norton theorem, the circuit in P5.24 becomes



where $\mathbf{V}_s = \mathbf{I}_s R$.

$$\therefore \quad \mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} \mathbf{I}_s$$

$$\therefore \quad \mathbf{H}(\omega) = \frac{\mathbf{I}}{\mathbf{I}_s}$$

$$= \frac{j\omega CR}{j\omega CR - \omega^2 LC + 1}$$

$$= \frac{j\omega CR}{(1-\omega^2 LC) + j\omega CR}$$

$$= \frac{\omega CR}{\sqrt{(1-\omega^2 LC)^2 + (\omega CR)^2}} \exp\left\{ j\left[90^\circ - \tan^{-1}\left(\frac{\omega CR}{1-\omega^2 LC}\right)\right]\right\}$$

Multiplying $\mathbf{H}(n\omega_0)$ with phasor representation of $\mathbf{I}_s$

$$\Rightarrow \quad \mathbf{I}(\omega) = \frac{A\tau}{T}\mathbf{H}(\omega = 0) + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right) \mathbf{H}(n\omega_0)$$

Note: $\mathbf{H}(\omega = 0) = 0$.

$$\mathbf{I}(\omega) = \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right) \frac{n\omega_0 CR}{\sqrt{(1-n^2\omega_0^2 LC)^2 + n^2\omega_0^2 (CR)^2}} \exp\left[j90^\circ - j\tan^{-1}\left(\frac{n\omega_0 CR}{1-n^2\omega_0^2 LC}\right)\right]$$

In time domain

$$i(t) = \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T}\right) \frac{n\omega_0 CR}{\sqrt{(1-n^2\omega_0^2 LC)^2 + n^2\omega_0^2 (CR)^2}} \cos\left[n\omega_0 t + 90^\circ - \tan^{-1}\left(\frac{n\omega_0 CR}{1-n^2\omega_0^2 LC}\right)\right]$$

(b) Substituting for $R = 1$ kΩ, $L = 1$ mH, $C = 1$ μF. The first five terms of $i(t)$ are:

$n = 1 \Rightarrow 0.0010\cos(\omega_0 t - 32^\circ)$

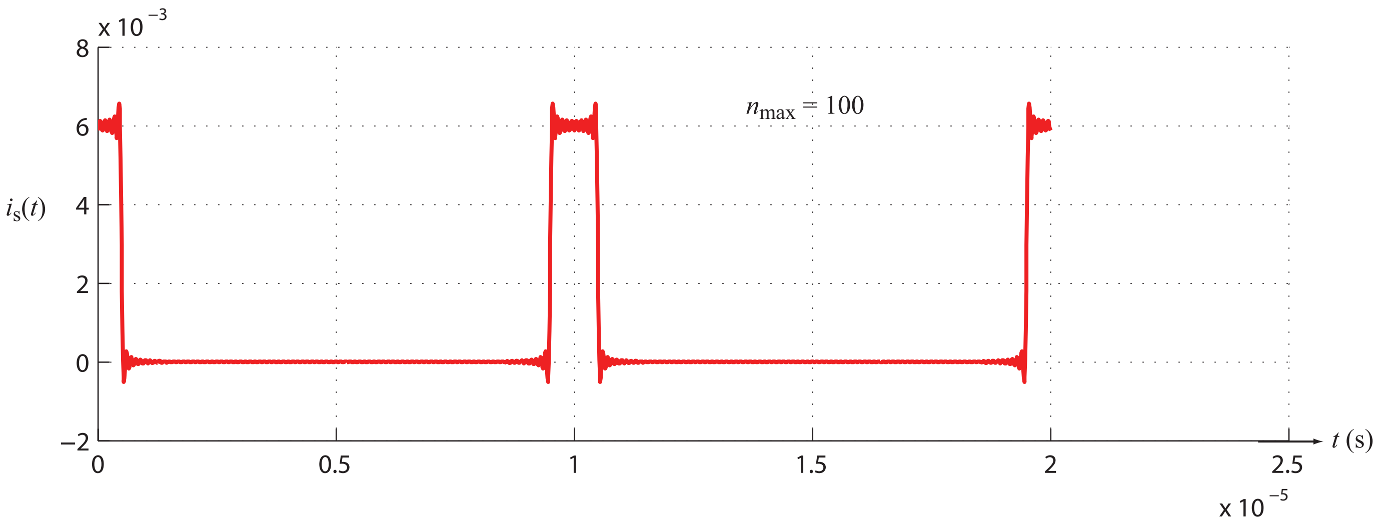
$n = 2 \Rightarrow 6.9928 \times 10^{-4}\cos(2\omega_0 t - 51.5^\circ)$

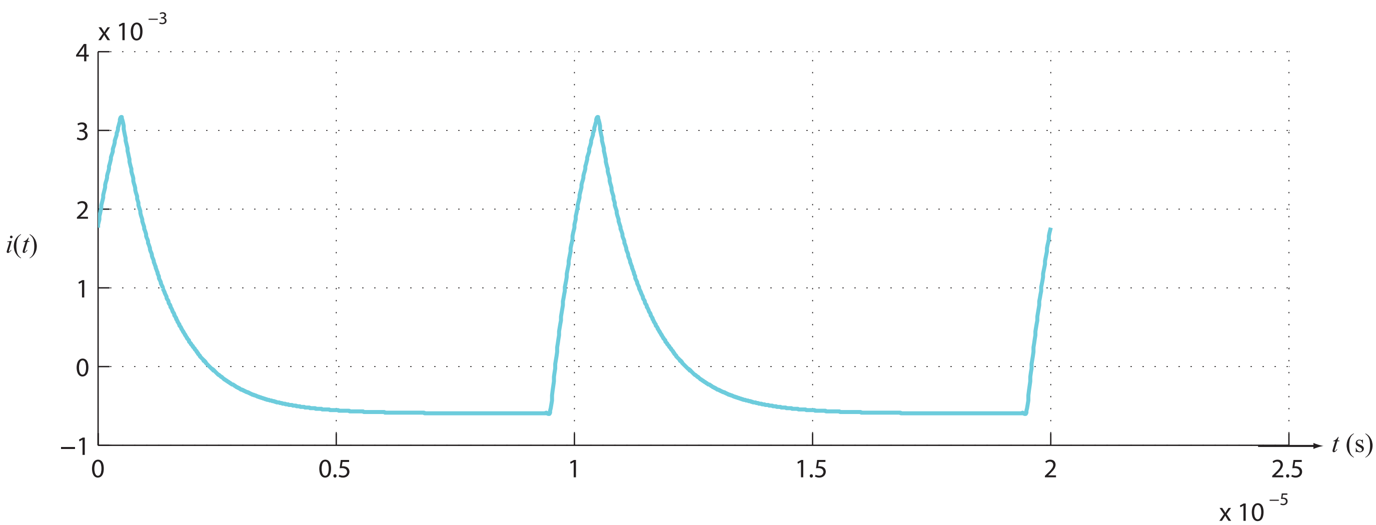
$n = 3 \Rightarrow 4.8285 \times 10^{-4}\cos(3\omega_0 t - 62^\circ)$

$n = 4 \Rightarrow 3.358 \times 10^{-4}\cos(4\omega_0 t - 68.3^\circ)$

$n = 5 \Rightarrow 2.3174 \times 10^{-4}\cos(5\omega_0 t - 72.3^\circ)$

(c)


x 10^−3
8
6
4
2
0
−2
$i_s(t)$
$n_{max} = 100$
0
0.5
1
1.5
2
2.5
t (s)
x 10^−5


x 10^−3
4
3
2
1
0
−1
$i(t)$
0
0.5
1
1.5
2
2.5
t (s)
x 10^−5