Problem 5.31 The circuit in Fig. P5.31 is excited by the source waveform shown in Fig. P5.26(b).

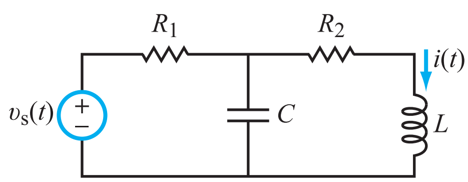


Figure P5.31: Circuit for Problem 5.31.

(a) Derive Fourier series representation of $i(t)$.

(b) Calculate the first five terms of $\upsilon_{\text{out}}(t)$ using $R_1 = R_2 = 100\ \Omega$, $L = 1$ mH, and $C = 1\ \mu$F.

(c) Plot $i(t)$ and $\upsilon_s(t)$ using $n_{\max} = 100$.

Solution: From Problem 5.25, the phasor domain presentation of $\upsilon_s(t)$ is

$$\mathbf{V}_s = 6 + \sum_{n=1}^{\infty} \frac{8[1-\cos(n\pi)]}{n\pi}\, e^{j90^\circ}.$$

Next, transfer function of circuit:

$$\text{define } \mathbf{Z}_1 = \frac{1}{j\omega C}, \qquad \mathbf{Z}_2 = j\omega L.$$

Then following the derivation in Problem 5.25,

$$\begin{aligned}
\mathbf{H} = \frac{\mathbf{I}}{\mathbf{V}_s} &= \frac{\mathbf{Z}_1}{(R_1+\mathbf{Z}_1)(\mathbf{Z}_1+\mathbf{Z}_2+R_2)-\mathbf{Z}_1^2} \\
&= \frac{\frac{1}{j\omega C}}{\left(R_1+\frac{1}{j\omega C}\right)\left(\frac{1}{j\omega C}+j\omega L+R_2\right)-\left(\frac{1}{j\omega C}\right)^2} \\
&= \frac{1}{(1+j\omega CR_1)\dfrac{(1-\omega^2LC+j\omega CR_2)}{j\omega C}-\dfrac{1}{j\omega C}} \\
&= \frac{j\omega C}{(1-\omega^2LC)+j\omega C[R_2+(1-\omega^2LC)R_1]-\omega^2C^2R_1R_2-1} \\
\mathbf{H}(\omega) &= \frac{j}{j[R_2+R_1(1-\omega^2LC)]-\omega(L+CR_1R_2)} \\
&= \frac{1}{\sqrt{\omega^2(L+CR_1R_2)^2+[R_2+R_1(1-\omega^2LC)]^2}} \\
&\quad \cdot \exp\left\{j90^\circ - \tan^{-1}\left[-\frac{R_2+R_1(1-\omega^2LC)}{\omega(L+CR_1R_2)}\right]\right\}
\end{aligned}$$

and

$$\mathbf{H}(\omega=0) = \frac{1}{R_1+R_2}.$$

The phasor domain presentation of the current **I**:

$$\mathbf{I}(\omega) = \frac{6}{R_1+R_2} + \sum_{n=1}^{\infty} \frac{8[1-\cos(n\pi)]}{n\pi} \frac{1}{\sqrt{n^2\omega_0^2(L+CR_1R_2)^2+[R_2+R_1(1-n^2\omega_0^2LC)]^2}}$$
$$\times \exp\left\{ j180^\circ - \tan^{-1}\left[-\frac{R_2+R_1(1-\omega^2LC)}{\omega(L+CR_1R_2)}\right]\right\},$$

and in time domain:

$$i(t) = \frac{6}{R_1+R_2} + \sum_{n=1}^{\infty} \frac{8(1-\cos n\pi)}{n\pi} \frac{1}{\sqrt{n^2\omega_0^2(L+CR_1R_2)^2+[R_2+R_1(1-n^2\omega_0^2LC)]^2}}$$
$$\times \cos\left(n\omega_0 t + 180^\circ - \tan^{-1}\left\{-\frac{[R_2+R_1(1-\omega^2LC)]}{\omega(L+CR_1R_2)}\right\}\right).$$

(b) Substituting the values of circuit components, the first five terms of $i(t)$ are

$n=1$ ➔ $0.0252\cos(\omega_0 t + 80^\circ)$

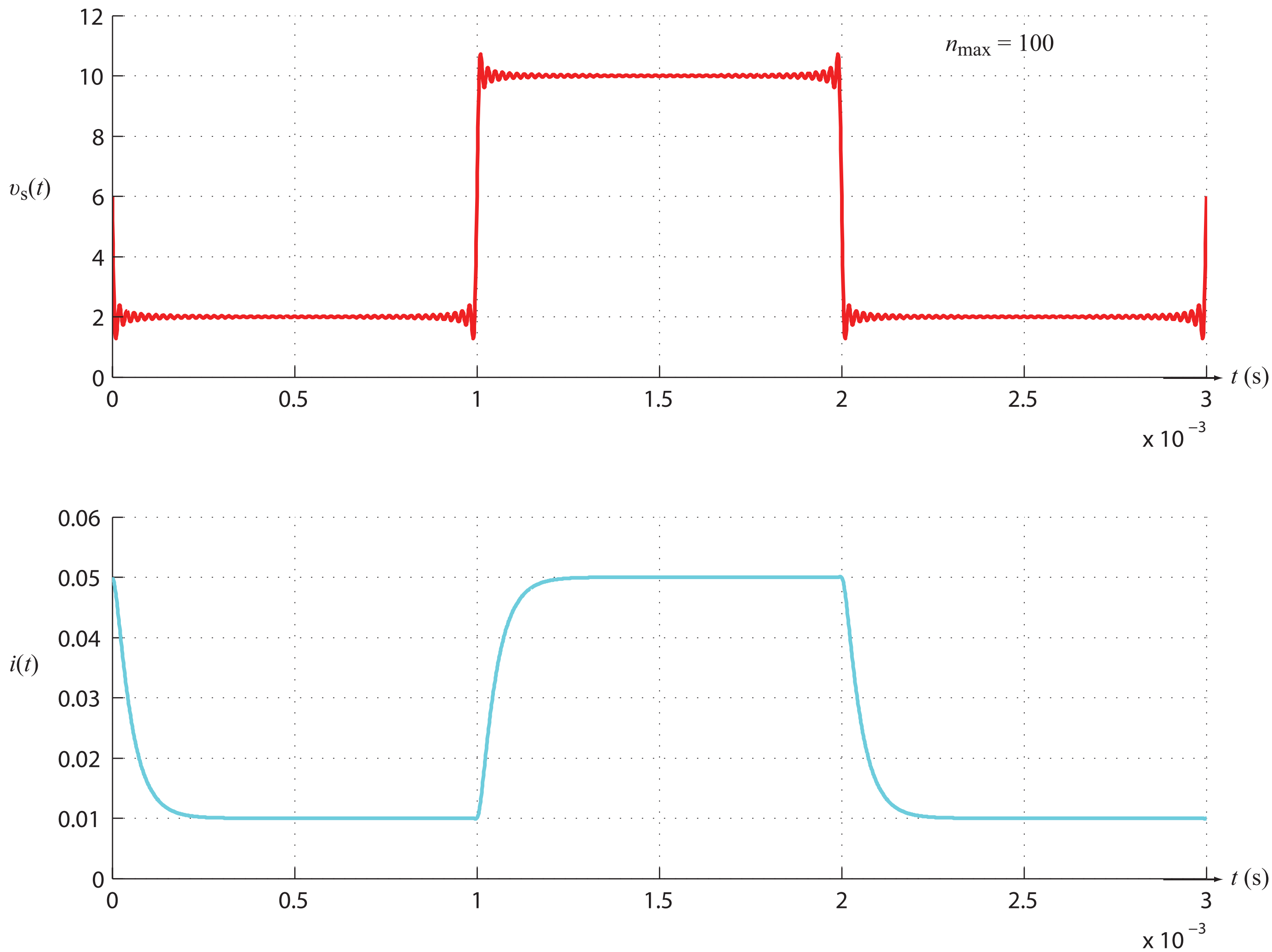
$n=2$ ➔ 0

$n=3$ ➔ $0.0078\cos(3\omega_0 t + 61.5^\circ)$

$n=4$ ➔ 0

$n=5$ ➔ $0.0041\cos(5\omega_0 t + 45.4^\circ)$

(c)



n_{max} = 100
$v_s(t)$
12
10
8
6
4
2
0
0.5
1
1.5
2
2.5
3
t (s)
x 10^{-3}
$i(t)$
0.06
0.05
0.04
0.03
0.02
0.01
0
t (s)
x 10^{-3}