Problem 5.9 Waveform in Fig. P5.9 with $A = 10$.

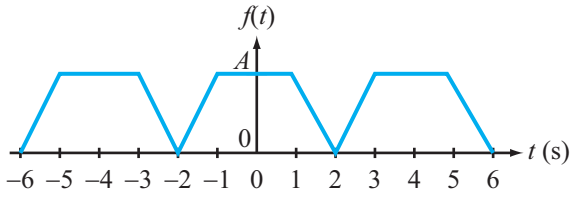


Figure P5.9: Waveform for Problem 5.9.

Solution:

(a) Waveform has even symmetry.

(b) Period $T = 4$ s

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2} \text{ rad/s}$$

$$f(t) = \begin{cases} 10(t+2) & -2 \le t \le -1 \\ 10 & -1 \le t \le 1 \\ -10(t-2) & 1 \le t \le 2 \end{cases}$$

$$a_0 = \frac{1}{4}\left[\int_{-2}^{-1} 10(t+2)\,dt + \int_{-1}^{1} 10\,dt + \int_{1}^{2} 10(2-t)\,dt\right]$$

$$= \frac{5}{2}\left[\left(\frac{t^2}{2}+2t\right)\Big|_{-2}^{-1} + t\Big|_{-1}^{1} + \left(2t-\frac{t^2}{2}\right)\Big|_{1}^{2}\right]$$

$$= \frac{15}{2}$$

$$a_n = \frac{2}{4}\left[\int_{-2}^{-1} 10(t+2)\cos\left(\frac{n\pi}{2}t\right)\,dt + \int_{-1}^{1} 10\cos\left(\frac{n\pi}{2}t\right)\,dt - \int_{1}^{2} 10(t-2)\cos\left(\frac{n\pi}{2}t\right)\,dt\right]$$

$$= 5\left\{\left[(t+2)\frac{\sin\left(\frac{n\pi}{2}t\right)}{(n\pi/2)}\right]\Big|_{-2}^{-1} - \int_{-2}^{-1}\frac{\sin\left(\frac{n\pi}{2}t\right)}{(n\pi/2)}\,dt + \frac{\sin\left(\frac{n\pi}{2}t\right)}{(n\pi/2)}\Big|_{-1}^{1}\right.$$

$$\left. - (t-2)\frac{\sin\left(\frac{n\pi}{2}t\right)}{(n\pi/2)}\Big|_{1}^{2} + \int_{1}^{2}\frac{\sin\left(\frac{n\pi}{2}t\right)}{(n\pi/2)}\,dt\right\}$$

$$= \frac{40}{(n\pi)^2}\left[\cos\left(\frac{n\pi}{2}\right) - \cos n\pi\right]$$

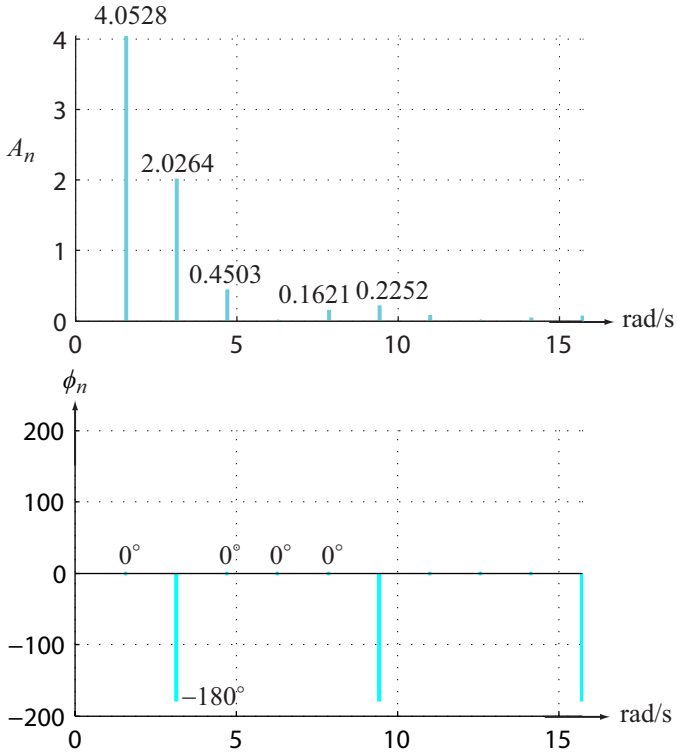
$$b_n = 0 \qquad \text{(even symmetry)}$$

$$f(t) = \frac{15}{2} + \sum_{n=1}^{\infty}\frac{40}{n^2\pi^2}\left[\cos\left(\frac{n\pi}{2}\right) - \cos n\pi\right]\cos\left(\frac{n\pi}{2}t\right)$$

(c) amplitude format

$$A_n = \sqrt{a_n^2 + b_n^2} = \frac{40}{n^2\pi^2}\left|\cos\left(\frac{n\pi}{2}\right) - \cos n\pi\right|$$

$$\phi_n = \begin{cases} 0 & \text{for } a_n > 0 \\ 180^\circ & \text{for } a_n < 0 \end{cases}$$



(d)