Problem 6.17 For the circuit shown in Fig. P6.17 provide the following:

(a) An expression for $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_i$ in standard form.

(b) Spectral plots for the magnitude and phase of $\mathbf{H}(\omega)$, given that $R = 10\ \Omega$, $L = 1$ mH, and $C = 10\ \mu$F.

(c) The cutoff frequency ω_c and the slope of the magnitude (in dB) when $\omega/\omega_c \ll 1$.

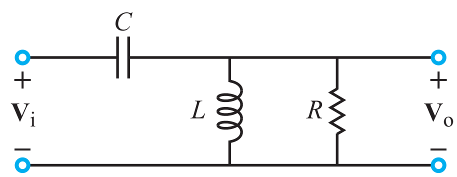


Figure P6.17: Circuit for Problem 6.17.

Solution:

(a) Voltage division yields

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{(R \parallel j\omega L)}{1/j\omega C + (R \parallel j\omega L)}$$
$$= \frac{-\omega^2 LC}{1 + j\omega L/R + j^2\omega^2 LC}$$
$$= \frac{-(\omega/\omega_c)^2}{1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2},$$

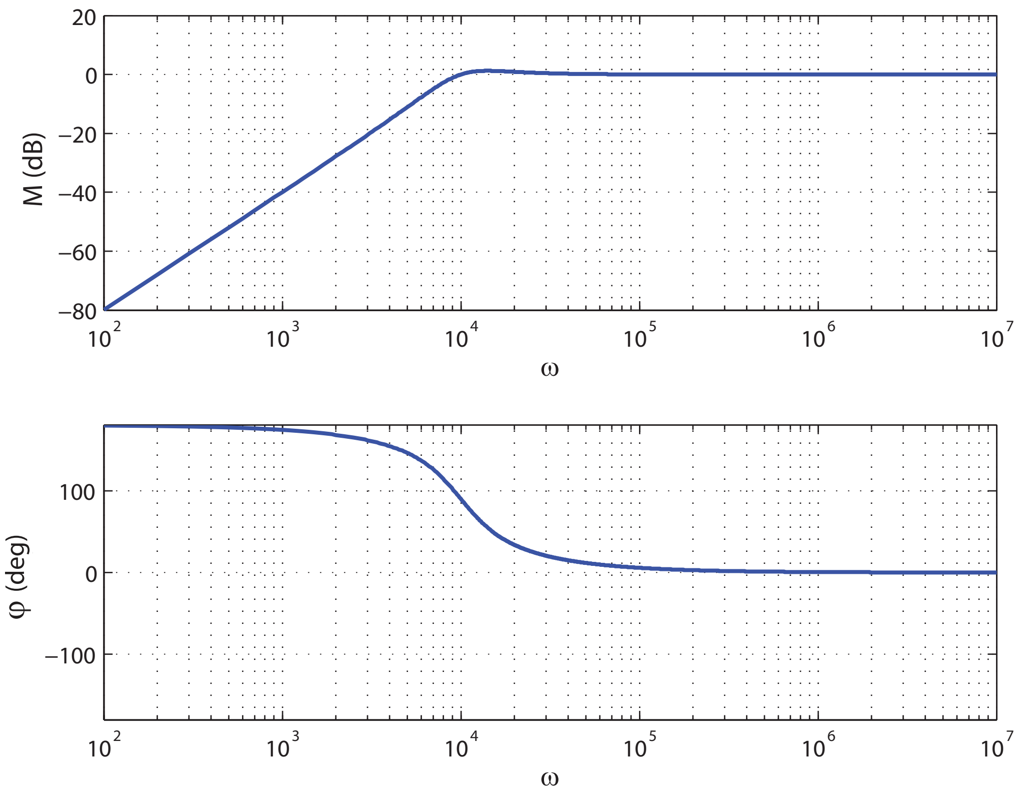
with

$$\omega_c = \frac{1}{\sqrt{LC}}, \qquad \xi = \frac{\omega_c L}{2R}.$$

(b) For $R = 10\ \Omega$, $L = 1$ mH, and $C = 10\ \mu$F,

$$\omega_c = 10^4 \text{ rad/s}, \qquad \xi = 0.5.$$

Spectral plots of M [dB] and $\phi(\omega)$ are shown in Figs. P6.17(a) and (b).



Figures P6.17(a) and (b)

(c) For $\omega/\omega_c \ll 1$,

$$\mathbf{H}(\omega) \approx -\left(\frac{\omega}{\omega_c}\right)^2 \qquad \Longrightarrow \qquad \text{slope} = +40 \text{ dB/decade}.$$