

Example 8-23: Computing CTFT by DFT.

Purpose:

Use the DFT to compute the Fourier transform of the continuous-time signals

$$(a) x_1(t) = e^{-|t|} \quad (b) x_2(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}.$$

For (a), we have $F[e^{-|t|}] = \frac{2}{\omega^2 + 1} = \frac{2}{4\pi^2 f^2 + 1}$
 $e^{-6} = 0.0025$, so $e^{-|t|} \approx 0$ for $|t| > 6$.
 $\frac{2}{4\pi^2 8^2 + 1} = 0.0008$, so $\frac{2}{4\pi^2 f^2 + 1} \approx 0$ for $|f| > 8$.

For (b), we have $F[\frac{e^{-t^2/2}}{\sqrt{2\pi}}] = e^{-\omega^2/2} = e^{-2\pi^2 f^2}$
 $\frac{e^{-4^2/2}}{\sqrt{2\pi}} = 0.00013$, so $\frac{e^{-t^2/2}}{\sqrt{2\pi}} \approx 0$ for $|t| > 4$.
 $e^{-2\pi^2 \cdot 0.6^2} = 0.00082$, so $e^{-2\pi^2 f^2} \approx 0$ for $|f| > 0.6$.

Inputs:

X = row vector of samples of $x(t)$.
 $t_{\max}$ = maximum time of $x(t)$ in s.
 $f_{\max}$ = maximum freq. of $X(j2\pi f)$ in Hz.

Outputs:

Actual 'o' and computed '+' spectra.

Comments:

- This uses the smallest possible DFT to compute CTFT samples accurately.
- The coarse discretizations of $\frac{1}{16}$ s and $\frac{1}{12}$ Hz work well for (a)!
- The *very* coarse discretizations of 0.8 s and 0.125 Hz work well for (b)!

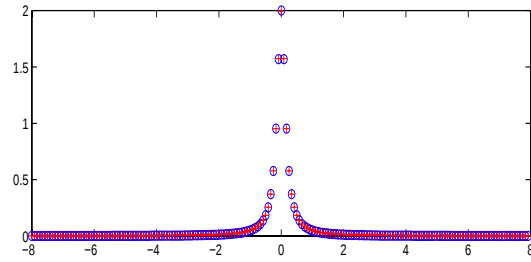


Figure 1: Example 8-23a: Actual 'o' and computed '+' spectrum values.

Program for (a):

```
clear;tmax=6;fmax=8;
T=2*tmax;B=2*fmax;
t=[-tmax:1/B:tmax-1/B];
f=[-fmax:1/T:fmax-1/T];
X=exp(-abs(t));
fX=2./(4*pi*pi*f.*f+1);
FX=fftshift(abs(fft(X)))/B;
subplot(211),
plot(f,fX,'o',f,FX,'+r')
```

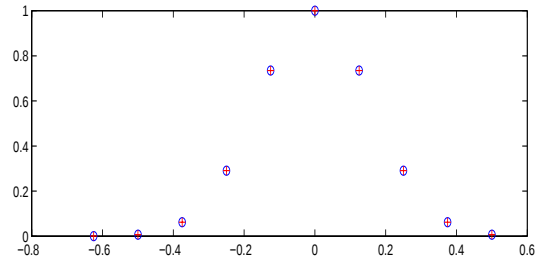


Figure 2: Example 8-23b: Actual 'o' and computed '+' spectrum values.

Program for (b):

```
clear;tmax=4;fmax=0.625;
T=2*tmax;B=2*fmax;
t=[-tmax:1/B:tmax-1/B];
f=[-fmax:1/T:fmax-1/T];
X=exp(-t.*t/2)/sqrt(2*pi);
fX=exp(-2*pi*pi*f.*f);
FX=fftshift(abs(fft(X)))/B;
subplot(211),
plot(f,fX,'o',f,FX,'+r')
```